Approximating Identity Conditions

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Criteria of Identity

- The introduction of the notion of identity criteria is attributed to Frege (*Grundlagen* §62):

  > *If we are to use symbol* **a** *to signify an object, we must have a criterion for deciding in all cases whether* **b** *is the same as* **a***, even if it is not always in our power to apply this criterion*

- A criterion of identity is a standard by which the identity of two items belonging to the same sort K is judged.
- Example: if a and b are lines, then the direction of line a is identical to the direction of line b iff a is parallel to b

Functions of Criteria of Identity

A criterion of identity seems to answer two questions[1]:

Ontological Question If *a* and *b* are Ks, what is for the object *a* to be identical to *b*?

Epistemic Question If *a* and *b* are Ks, how can we know that *a* is the same as *b*?

[1] Carrara, M. and Giaretta, P., The Many Facets of Identity Criteria. *Dialectica* 58(2), 2004

Formulations of Criteria of Identity

- Formulation of a criterion of identity:

$$\forall x \forall y((K(x) \wedge K(y)) \rightarrow (x = y \leftrightarrow \Phi(x, y))) \qquad \text{(IC)}$$

- Φ represents the identity condition (the standard under which x and y are identical)
- Left side of biconditional: $x = y$ is an equivalence relation $\Rightarrow$ right side: there must be an equivalence relation R

Failure of Transitivity

Relations considered as intuitively good identity conditions often do not meet the logical requirement that IC demands: transitivity fails. Some examples from Williamson[2]:

- Let $x, y, z, \ldots$ range over colour samples. The colour of x is identical to the colour of y iff x and y are indistinguishable in colour
- Let $x, y, z, \ldots$ be physical magnitudes. $x = y$ iff x and y turn out to be the same under some measurement

How to get logical adequacy?

[2] Williamson, T., Criteria of Identity and the Axiom of Choice. *The Journal of Philosophy* 83, 1986.

Williamson's Approaches

- Give up the requirement for the identity condition to be both necessary and sufficient;
- Given a non transitive R, let $R', R'', \ldots$ be equivalence relations that approximate R;
- Find the best approximation R' in either of two ways:

Approach from above
: Consider the smallest (unique) equivalence relation R^+ s. t. $R \subseteq R^+$ (sufficient condition)

Approach from below
: Consider the largest (not unique) equivalence relation R^- s. t. $R^- \subseteq R$ (necessary condition)

De Clercq and Horsten's Option

- Given a kind of objects K, there are not always good reasons to decide whether you must take a necessary or a sufficient condition
- Third option: giving up both the necessity and the sufficiency of the identity condition
- To seek for an overlapping relation $R^{\pm}$ (neither a super- nor a sub-relation of R)

(De Clercq, R. and Horsten, L., Closer. *Synthese* 146(3), 2005)

Overlapping Approach 1

- Consider an example. Let $\mathcal{D} = \{a, b, c, d, e\}$ be a domain of objects
- Let R be a given relation, reflexive and symmetric
- If R holds between two elements x, y, write the pair as follows: $\overline{xy}$
- Let R on $\mathcal{D}$ be the following: $R = \{\overline{ac}, \overline{ad}, \overline{bc}, \overline{bd}, \overline{cd}, \overline{de}\}$
- R is not an equivalence relation (R holds between a and d and between d and e, but it does not hold between a and e)

Overlapping Approach 2

How to approximate $R = \{\overline{ac}, \overline{ad}, \overline{bc}, \overline{bd}, \overline{cd}, \overline{de}\}$?

Approach from above Add fours pairs:

$$R^{+} = \{\overline{ab}, \overline{ac}, \overline{ad}, \overline{ae}, \overline{bc}, \overline{bd}, \overline{be}, \overline{cd}, \overline{ce}, \overline{de}\}$$

Approach from below Remove three pairs:

$$R^{-} = \{\overline{bc}, \overline{bd}, \overline{cd}\}$$

Overlapping Approach Add and remove one pair:

$$R^{\pm} = \{\overline{ab}, \overline{ac}, \overline{ad}, \overline{bc}, \overline{bd}, \overline{cd}\}$$

Overlapping Approach 3

Which is the best equivalence approximation of a non transitive relation R?

- Call *revision* any adding or removing of a pair to or from R
- Let the degree of unfaithfulness of an approximation R' be the number of revisions you make to get R' from R
- Given two approximations R', R'', R' is closer to R than R'' iff the degree of unfaithfulness of R' is lower than the degree of unfaithfulness of R''
- In the example above, R^+ has degree of unfaithfulness of 4, R^- of 3, $R^\pm$ of 2;
- $R^\pm$ is closer to R than R^+ and R^-.

How to Choose R?

- De Clercq and Horsten do not discuss how we should choose the candidate relation R. Sometimes they claim that such an R is an obvious candidate. But from which point of view do you consider such a relation obvious? How can we choose the best candidate R for some objects of sort K?
- It is worthy to make some considerations on the conditions that R must meet in order to be a plausible candidate for being an identity condition.

Assumption concerning R

- De Clercq and Horsten assume that, given a relation R for objects of kind K and given two objects x and y belonging to K, either R holds between x and y or it does not hold
- But if R holds between two objects a and b, can there be some situations where R does not hold between a and b?

Problematic Situations 1

Example a You see two mono-chromatic spots, A and B, and you do not detect any difference with respect of their colour. You say that they have the same colour. Now, you get a colour spectrum and compare A and B with it. You notice that they correspond to two spots of the spectrum that are not contiguous. You revise your judgement and say that A and B are distinct.

Contexts

- Example **a** shows that our judgements about colours depend on how we compare colour samples
- R can vary across contexts
- Two objects that are indistinguishable in a context, and therefore judged as identical, can turn out to be distinct in another context
- However, the relation of identity is maintained absolute in each context

Problematic Situations 2

Example b You see two colour samples A and B from a distant point of view such that you are not able to distinguish A-colour from B-colour. You say that they have the same colour. Now you get closer to them and detect a difference between them. So, you revise your previous judgement and say that A and B are distinct

Example c You see two spots, A and B, and you perceive them as equally, say, orange. A painter tells you that (s)he perceives them distinct: A is more yellowish than B

Granular Levels

- Examples **b** and **c** present a different issue. A context is fixed and R varies along different levels of observation
- From a distant, coarse point of view, you make an identity statement about some objects x, y in a context o via R: for instance, $x = y$
- From a more precise, fine-grained point of view, you can make a different identity statement about the same objects x, y in o via R: for instance, $x \neq y$
- You can look at the elements of a context under different standards of precision (granular levels). Finer the level is, more differences between the individuals can be detected

Language 1

Let $\mathcal{L}$ be a formal language through which we can represent English expressions. $\mathcal{L}$ consists of:

- individual constant symbols: $\overline{a}, \overline{b}, \ldots$ (there is a constant symbol for each element of the domain);
- individual variable symbols: $x_0, x_1, x_2, \ldots$ (countably many);
- 2-arity predicate symbols $P_1, P_2, \ldots$;
- usual logical connectives with identity, quantifiers.

Language 2

The set of terms consists of individual constant and individual variable symbols.
Formulas are defined as follows:

- If t_1, t_2 are terms, then $P_1(t_1, t_2), P_2(t_1, t_2), \ldots$ are formulas;
- If t_1, t_2 are terms, then $t_1 = t_2$ is a formula;
- If ϕ, ψ are formulas, then $\phi \Box \psi$ is a formula, where $\Box$ is one of the usual logical connectives;
- If ϕ is a formula, then $\neg\phi$ is a formula;
- If ϕ is a formula, then $\forall x_i \phi, \exists x_i \phi$ are formulas.

Context Structures

- Let $\mathcal{M} = \langle \mathcal{D}, R \rangle$ be a fixed model or context structure, consisting of a fixed, non empty domain $\mathcal{D}$, and a binary relation R
- Each subset of the domain $\mathcal{D}$ is a context o. The set of all contexts O is the powerset of $\mathcal{D}$: $O = \wp(\mathcal{D})$
- R is reflexive and symmetric, non necessarily transitive

Behaviour of R

- Given a context structure $\mathcal{M}$, R can vary across contexts
- Given a $\mathcal{M}$, if R fails to be transitive with respect to some (if not all) contexts $o \in O$, for each of those o an equivalence overlapping relation $R^{\pm}$ can be defined
- Context structures can belong to different granular levels
- Given a context $o \in O$, different context structures can give different sets of pairs generated by R
- We can partially order the context structures from the coarsest to the finest with respect to a context $o \in O$

Example

- Let $o = \{a, b, c, d, e\}$ be a given context
- Consider two context structures, $\mathcal{M}_1$, $\mathcal{M}_2$
- $\mathcal{M}_1$ gives $R = \{\overline{ab}, \overline{bc}, \overline{de}\}$. The best approximation is $R^{\pm} = \{\overline{ab}, \overline{bc}, \overline{ac}, \overline{de}\}$
- $\mathcal{M}_2$ gives $R = \{\overline{ab}, \overline{bc}, \overline{cd}, \overline{de}, \overline{ce}\}$. The best approximation is $R^{\pm} = \{\overline{cd}, \overline{de}, \overline{ce}\}$

Conclusion

- Before determining the closest approximation to R we suggest to fix a context and a granular level of observation
- R can vary across contexts and granular levels
- If according to a context structure R fails to be transitive in a context, you can build the closest approximation to R for that context and context structure

Thank you

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