PhD's in Logic

Syntactic Characteristics of Empirical Substructures

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The Received View

- Theory given as a set of sentences Σ.
- Sentences in first order logic (historically questionable).
- Bipartition of the vocabulary $\mathscr{V}$ into $\mathscr{O}$-terms and $\mathscr{T}$-terms.
- $\Sigma = T \cup C$
  - T a set of $\mathscr{T}$-sentences.
  - C a set of $\mathscr{V}$-sentences.
- Observations given as a set Ω of $\mathscr{O}$-sentences.
- Sometimes: Σ explains the observations iff $\Sigma \vdash \Omega$ (Hempel–Oppenheim-Schema).

Van Fraassen's critique of the Received View

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Two readings:

1. Against syntactic relations.

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Two readings:

1. Against syntactic relations.
2. Against the $\mathscr{O}/\mathscr{T}$-bipartition.

Van Fraassen's Constructive Empiricism

- A theory is given by a class $\mathbf{T}$ of theory structures closed under isomorphism.
- Each theory structure $\mathfrak{T} \in \mathbf{T}$ has an empirical substructure $\mathfrak{T}^{\mathrm{e}}$.
- Appearances are given by an observational structure $\mathfrak{O}$.
- $\mathbf{T}$ is empirically adequate iff there is a $\mathfrak{T} \in \mathbf{T}$ such that $\mathfrak{T}^{\mathrm{e}} \cong \mathfrak{O}$.

Note:

- Since $\mathbf{T}$ is closed under isomorphism, $\mathfrak{O} \cong \mathfrak{T}^{\mathrm{e}}$ for some $\mathfrak{T} \in \mathbf{T}$ iff $\mathfrak{O} = \mathfrak{T}^{\mathrm{e}}$ for some $\mathfrak{T} \in \mathbf{T}$.

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If not: Make a new class of structures $\mathbf{T}'$ with a predicate O such that for each $\mathfrak{T}$, $\mathfrak{T}|_O = \mathfrak{T}^{\mathrm{e}}$.

$\mathfrak{T}|_O$ is the substructure resulting from the restriction of $\mathfrak{T}$'s domain to $O^{\mathfrak{T}}$.

Classes of structures and sets of sentences

If a theory is given by a set of sentences Σ:

$$\mathbf{T}_\Sigma := \{\mathfrak{A} : \mathfrak{A} \models \Sigma\}$$

If a theory is given by a class of structures $\mathbf{T}$ that is closed under elementary equivalence, there is a set of sets $\{\Sigma_i\}_{i \in I}$ such that

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- Assume that theory and observations are correctly described in either view.
- Assume that $\{\Sigma_i\}_{i\in I} = \{\Sigma_\mathbf{T}\}$ ($\Rightarrow \Sigma_\mathbf{T} = \bigcap_{\mathfrak{T}\in\mathbf{T}} \mathrm{Th}\mathfrak{T}$).

Syntactic features of theories and observations

For all theories it holds that

$$\exists x Ox$$

$$Ox_1 \wedge \cdots \wedge Ox_n \rightarrow Of x_1 \ldots x_n \text{ for all } f$$

$$Oc \text{ for all constants } c$$

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$$\begin{aligned} (\forall x \varphi)^O &:= \forall x (Ox \rightarrow (\varphi)^O \\ (\exists x \varphi)^O &:= \exists x (Ox \wedge (\varphi)^O \\ (\varphi \wedge \psi)^O &:= (\varphi)^O \wedge (\psi)^O \\ (Px)^O &:= Px \end{aligned}$$

$$\Sigma|_O := \{\varphi : \varphi \models (\varphi)^O, \Sigma \vDash \varphi\}$$

Syntactic characterization of empirical adequacy

By the relativization theorem (Hodges, theorem 5.1.1):
there is a $\mathfrak{T} \in \mathbf{T}_\Sigma$ with $\mathfrak{O} \equiv \mathfrak{T}^e$ iff $\Sigma|_O$ is compatible with $\Omega_\mathfrak{O} := \mathrm{Th}\mathfrak{O}$.

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$\mathbf{T}_\Sigma$ is empirically adequate for some $\mathfrak{A} \models \Omega_\mathfrak{O}$ iff Σ is compatible with $(\Omega_\mathfrak{O})^O$.
Note:

- Empirical adequacy means compatibility with the observations, not entailment of the observations.
- If Σ is axiomatizable, then so is $\Sigma|_O$ (by Craig's theorem).
- Open question: Is $\mathbf{T}_\Sigma$ empirically adequate for $\mathfrak{O}$ iff $\Sigma|_O$ is compatible with $\Omega_\mathfrak{O}$?

A rational reconstruction of $\Sigma_{\mathbf{T}}$ *for* $\mathscr{O}/\mathscr{T}$*-empirical adequacy*

$\mathbf{T}_\Sigma$ is empirically adequate iff

$$\mathfrak{O} \in \{\mathfrak{T}|_O : \mathfrak{T} \models \Sigma\}$$

Hodges (theorem 5.2.1) shows that Σ has a reformulation Σ' such that

$$\{\mathfrak{T}|_O : \mathfrak{T} \models \Sigma\} = \{\mathfrak{T}|_{\mathscr{O}} : \mathfrak{T} \models \Sigma'\}$$

$\mathfrak{T}|_{\mathscr{O}}$ is the reduct of $\mathfrak{T}$ to $\mathscr{O}$.

A cheap trick for $\mathscr{O}/\mathscr{T}$-empirical adequacy

For a theory Σ in $\mathscr{V}$

- define $\mathscr{O} := \mathscr{V}$,
- define $\mathscr{T} := \{X' : X \in \mathscr{O}\}$,
- define $C := \{\forall x(Px \leftrightarrow P'x), \forall x fx = f'x, c = c', \text{etc.}\}$
- define T as the set of $\mathscr{T}$-sentences entailed by $\Sigma \cup C$: $T := \mathrm{Cn}(\Sigma \cup C)|\mathscr{T}$.

Then $\mathbf{T}_\Sigma$ is empirically adequate for some $\mathfrak{A} \vDash \Omega_\mathfrak{H}$ iff $T \cup C$ is compatible with $(\Omega_\mathfrak{H})^O$.
Constructive Empiricism as a special kind of Received View (with restrictions on the observation sentences).

Using C for $\mathscr{O}/\mathscr{T}$-empirical adequacy

- define $\mathscr{O} := \mathscr{V}$,
- define $\mathscr{T} := \{X' : X \in \mathscr{O}\}$,
- define T as before as Σ with primed non-logical constants,
- define
  $C := \{\forall x[Ox \rightarrow (Px \leftrightarrow P'x)], \forall x(Ox \rightarrow fx = f'x), c = c', \text{etc.}\}$

Then $\mathbf{T}_\Sigma$ is empirically adequate for some $\mathfrak{A} \equiv \mathfrak{O}|_O$ iff $T \cup C$ is compatible with $\Omega_\mathfrak{O}$.

Conclusion

Van Fraassen's semantic conception of the relation between theory and observation can be captured

syntactically with the help of the relativization theorem, and

in the Received View in at least three ways:

- via a reformulation of the theory,
- by doubling the vocabulary and relativizing observation sentences,
- by doubling the vocabulary and defining theoretical terms conditionally (note: requires a restriction of $\mathfrak{D}$ to O).

Vindication of the Received View.
Contribution to Constructive Empiricism.